



21

P. مود الامتحان

Circuit2
First

Circuit2

30/03/2011

90 min

اسم المساق :

تاريخ الامتحان :

زمن الامتحان :

الشعبة : اثنين اربعاء 8-9:30

الاسم : معن سلام قعدان

العلامة

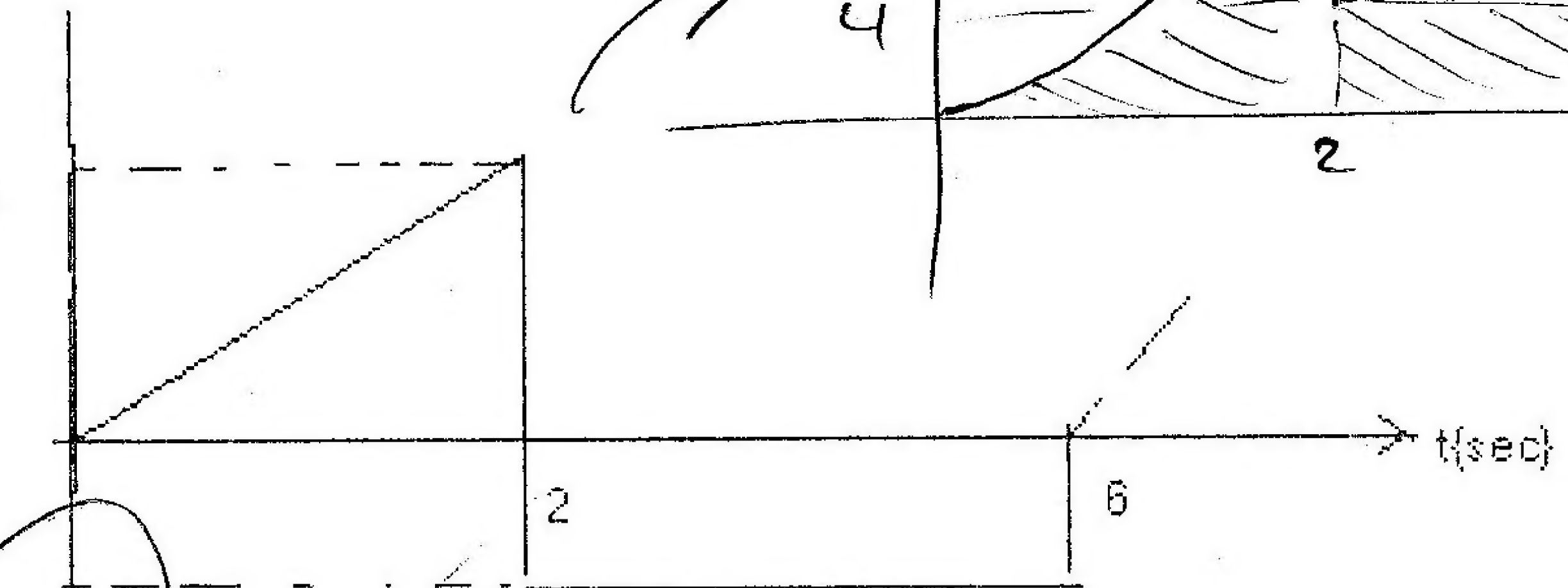
21
30

Q1) a) Find the RMS value of the following periodic waveform

(3)

$$I_{eff} = \frac{i(t)}{\sqrt{2}} = \sqrt{\frac{i^2(t)}{2}}$$

i(t)



$$i(t) (0 < t < 2)$$

$$= 2t$$

$$i(t) (2 < t < 6)$$

$$= -2$$

$$\Rightarrow i^2(t) (0 < t < 2) = 4t^2$$

$$\Rightarrow i^2(t) (2 < t < 6) = 4$$

$$\begin{aligned} I_{eff} &= \sqrt{\frac{1}{6} \int_0^6 i^2(t) dt} = \sqrt{\frac{1}{6} \left(\int_0^2 4t^2 dt + \int_2^6 4 dt \right)} \\ &= \sqrt{\frac{1}{6} \left(\frac{4t^3}{3} \Big|_0^2 + 4t \Big|_2^6 \right)} = \sqrt{\frac{1}{6} \left(\frac{32}{3} + 16 \right)} = \sqrt{\frac{1}{6} \left(\frac{80}{3} \right)} = \sqrt{\frac{40}{9}} = \frac{2\sqrt{10}}{3} \end{aligned}$$

$$\begin{aligned} I_{eff} &= \sqrt{\frac{1}{6} \int_0^6 i^2(t) dt} = \sqrt{\frac{1}{6} \left(\int_0^2 4t^2 dt + \int_2^6 4 dt \right)} \\ &= \sqrt{\frac{1}{6} \left(\frac{4t^3}{3} \Big|_0^2 + 4t \Big|_2^6 \right)} = \sqrt{\frac{1}{6} \left(\frac{32}{3} + 16 \right)} = \sqrt{\frac{1}{6} \left(\frac{80}{3} \right)} = \sqrt{\frac{40}{9}} = \frac{2\sqrt{10}}{3} \end{aligned}$$

$$\therefore I_{eff} = \frac{4}{\sqrt{2}} + 4\sqrt{2} = 8.465 \text{ A}$$

Ieff total

b) Find the average value of the following periodic waveform (3)

$$V_{avg} = \frac{1}{T} (\text{area under the curve})$$

v [volts]

area ① :-

$$\begin{aligned} \int_0^{\pi} \sin t dt &= -\cos t \Big|_0^{\pi} \\ &= -\cos \pi - (-\cos 0) \\ &= (-1) - (-1) = 1 + 1 = 2 \end{aligned}$$

$$\text{area ②} : \frac{1}{2} (2\pi - \pi) (3) = \frac{1}{2} \pi (3) = \frac{3\pi}{2}$$

$$= \frac{1}{2} \pi (3) = \frac{3\pi}{2}$$

$$\text{area ③} :- (3\pi - 2\pi) (2) = 2\pi$$

$$\therefore \text{Area under the curve} = 2 + \frac{3\pi}{2} + 2\pi = \frac{4}{2} + \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{4 + 7\pi}{2}$$

$$\therefore V_{avg} = \frac{1}{3\pi} \left(\frac{4 + 7\pi}{2} \right) = \frac{1}{3\pi} \left(\frac{4 + 7\pi}{2} \right) = \frac{4 + 7\pi}{6\pi} = \frac{2}{3\pi} + \frac{7}{6}$$

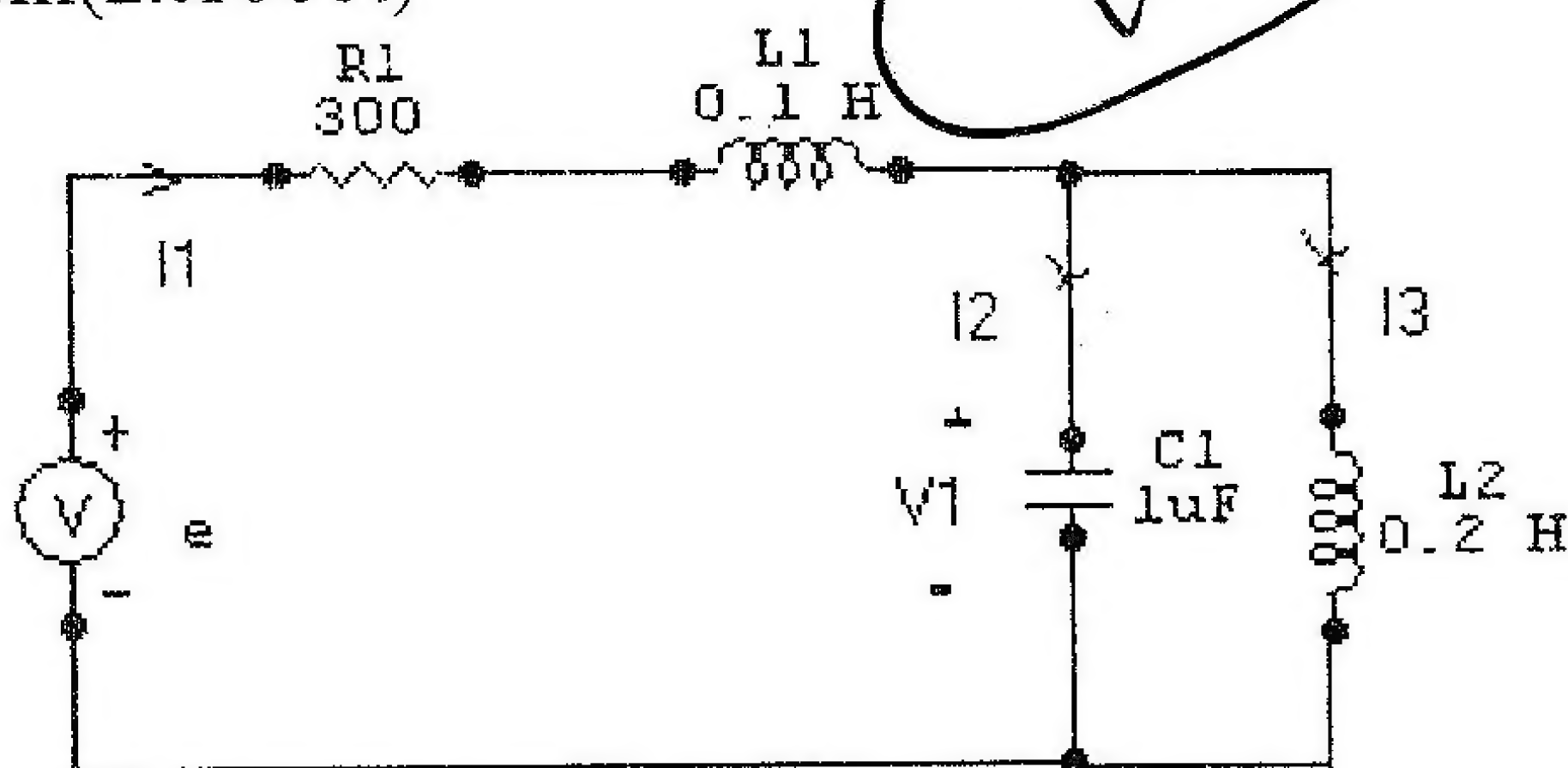
Page 1

تم ارفعه بواسطة
P. معن ابو عيسى

Q2) In the circuit shown below if $e = 50\sin(2\pi 1000t)$

$$e = \frac{50}{\sqrt{2}} \angle 0^\circ$$

$$= 35.35 \angle 0^\circ$$



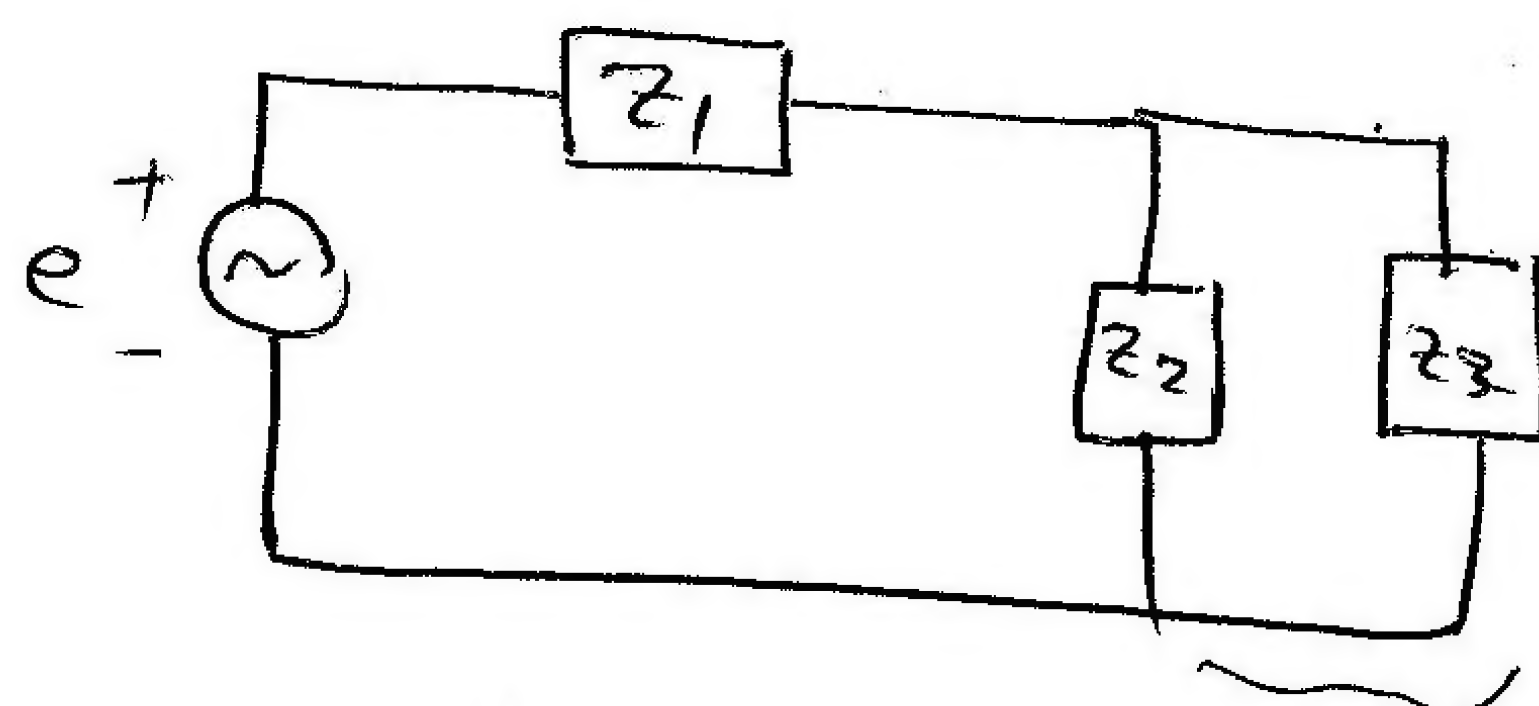
$$P_{avg} = VI \cos(\theta_v - \theta_i)$$

- Find the total impedance (Z_T) and total admittance (Y_T) (2.5)
- Find the total current (I_1) (1)
- Find $i_3(t)$, $i_2(t)$ and $v_1(t)$ (3)
- Find the average power delivered to the circuit (1.5)
- Find the power factor of the circuit and indicate whether it leading or lagging (1)

$$X_{L1} = \omega L_1 = 2\pi \times 1000 (0.1) = 628.3 \Omega$$

$$X_{L2} = \omega L_2 = 2\pi \times 1000 (0.2) = 1256.6 \Omega$$

$$X_{C1} = \frac{1}{\omega C} = \frac{1}{2\pi \times 1000 \times 1 \times 10^{-6}} = 159 \Omega$$



$$Z_1 = (300 + j628)$$

$$Z_2 = -j159$$

$$Z_3 = j1256$$

$$Z_2 \parallel Z_3 = \frac{-j159 \times j1256}{1256j - 159j} = \frac{199704}{1097j} = 182 \angle -90^\circ = -j182$$

$$\therefore Z_T = Z_1 + (Z_2 \parallel Z_3) = 300 + j628 - j182 = 300 + j446$$

$$\therefore Z_T = 537.5 \angle 56^\circ$$

$$\therefore Y_T = \frac{1}{537.5 \angle 56^\circ} = 0.00186 \angle -56^\circ \text{ Siemens}$$

$$b) I_T = \frac{E}{Z_T} = \frac{35.35 \angle 0^\circ}{537.5 \angle 56^\circ} = 0.065 \angle -56^\circ \text{ A} = I_1$$

$$c) I_2 = I_1 \frac{j1256.6}{j1256.6 - j159} = 0.065 \angle -56^\circ \left[\frac{j1256.6}{j1097} \right] = 0.074 \angle -56^\circ \text{ A}$$

$$i_2(t) = 0.074(\sqrt{2}) \sin(2\pi \times 1000t - 56^\circ) = 0.104 \sin(2\pi \times 1000t - 56^\circ) \text{ A}$$

$$I_3 = I_1 \frac{j159}{j1256 - j159} = I_1 \frac{159 \angle -90^\circ}{1097 \angle -90^\circ} = 0.065 \angle -56^\circ \left[0.145 \angle -180^\circ \right] = 0.00942 \angle -236^\circ \text{ A}$$

$$i_3(t) = 0.00942 \sqrt{2} \sin(2\pi \times 1000t - 236^\circ) = 0.0133 \sin(2\pi \times 1000t - 236^\circ) \text{ A}$$

$$V_1 = I_2 Z_2 = 0.074 \angle -56^\circ \times 159 \angle -90^\circ = 11.766 \angle -146^\circ \text{ V}$$

$$v_1(t) = 11.766 \sqrt{2} \sin(2\pi \times 1000t - 146^\circ) = 16.63 \sin(2\pi \times 1000t - 146^\circ) \text{ V}$$

$$d) P_{avg} = I V \cos(\theta_v - \theta_i)$$

~~$$= 0.065 \times 56 \times 35.35 / 60 \cos(0 - 56)$$~~

$$= 0.065 \times 35.35 \cos(0 - 56)$$

$$= 0.065 \times 35.35 \cos 56^\circ$$

$$= \underline{\underline{1.284 \text{ Watt}}}$$

$$e) F_p = \cos(\theta_v - \theta_i) = \cos 56^\circ = \boxed{0.56}$$

It's lagging Power factor

~~use~~ Inductive Network

Q3) Find the element or elements that must be in the closed box in the circuit to satisfy the following

Conditions:

1) Average power to the circuit is 3000 W

2) Circuit has a leading power factor

① $P_{avg} = I V \cos \theta$

~~$3000 = I V \cos \theta$~~

$$\therefore \cos \theta = \frac{P_{avg}}{I V} = \frac{3000}{40(100)} = \frac{3000}{4000} = 0.75$$

$$\therefore \theta = \cos^{-1}(0.75) = 41.4^\circ$$

② \Rightarrow leading Power factor

$$\Rightarrow \therefore I = 40 \angle 41.4^\circ \text{ A}$$

\therefore Capacitive Network

\therefore The element must be a capacitor

$$\therefore Z_T = \frac{V_T}{I_T} = \frac{100 \angle 0^\circ}{40 \angle 41.4^\circ} = 2.5 \angle -41.4^\circ \Omega$$

Capacitor

$$Z_T = \frac{20 Z_1}{20 + Z_1}$$

$$= 2.5 (\cos 41.4^\circ - j \sin 41.4^\circ)$$

$$= 2.5 (0.75 - j 0.66) = 1.875 - j 1.65$$

$$\therefore 1.875 - j 1.65 = \frac{20 Z_1}{20 + Z_1}$$

$$(1.875 - j 1.65)(20 + Z_1) = 20 Z_1$$

$$37.5 + 1.875 Z_1 - j 33 - j 1.65 Z_1 = 20 Z_1$$

$$(37.5 - j 33) + (1.875 Z_1 - j 1.65 Z_1) = 20 Z_1$$

Q4) Using superposition find the sinusoidal expression for the voltage V_c in the circuit below (4)

Using Superposition

① From 12 V Source

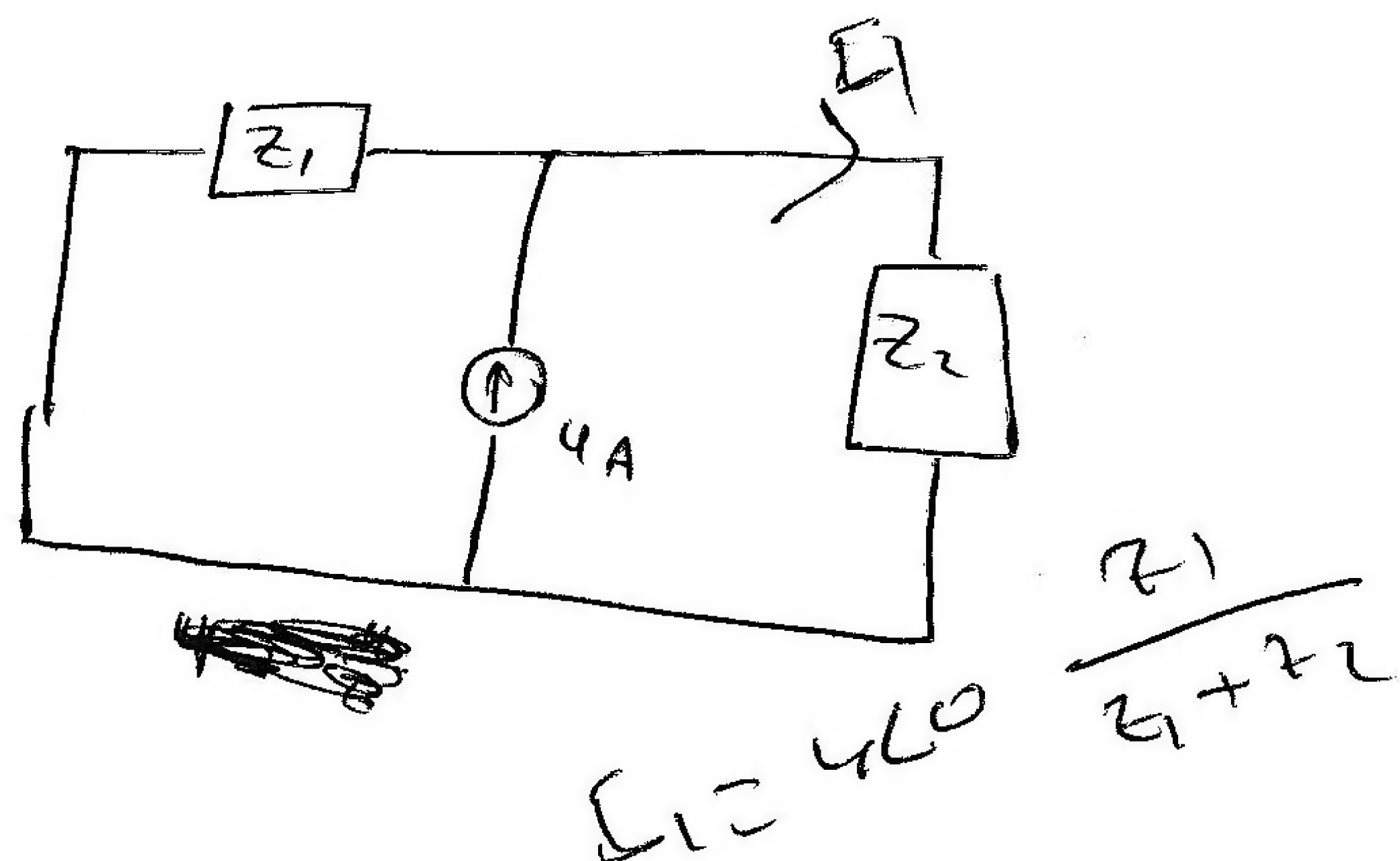
$$V_{c1} = 12 \text{ V}$$

② From 4 A Source

$$Z_1 = 6 \Omega = 6 \angle 0^\circ$$

$$Z_2 = 3 - j2$$

$$= 3.6 \angle 33.6^\circ$$



$$\therefore V_{c2} = I_2 \times Z_2$$

$$= 4 \angle 0^\circ \times 3.6 \angle 33.6^\circ = 14.4 \angle 33.6^\circ \text{ V}$$

$$\therefore V_c = V_{c1} + V_{c2} = 12 \text{ V} + 14.4 \angle 33.6^\circ \text{ V}$$

$$= 12 + 14.4 (\cos 33.6^\circ + j \sin 33.6^\circ)$$

$$= 12 + 14.4 (0.83 + j0.55)$$

$$= 12 + 11.952 + j7.92$$

$$= 23.952 + j7.92 = 25.2 \angle 18.2^\circ \text{ V}$$

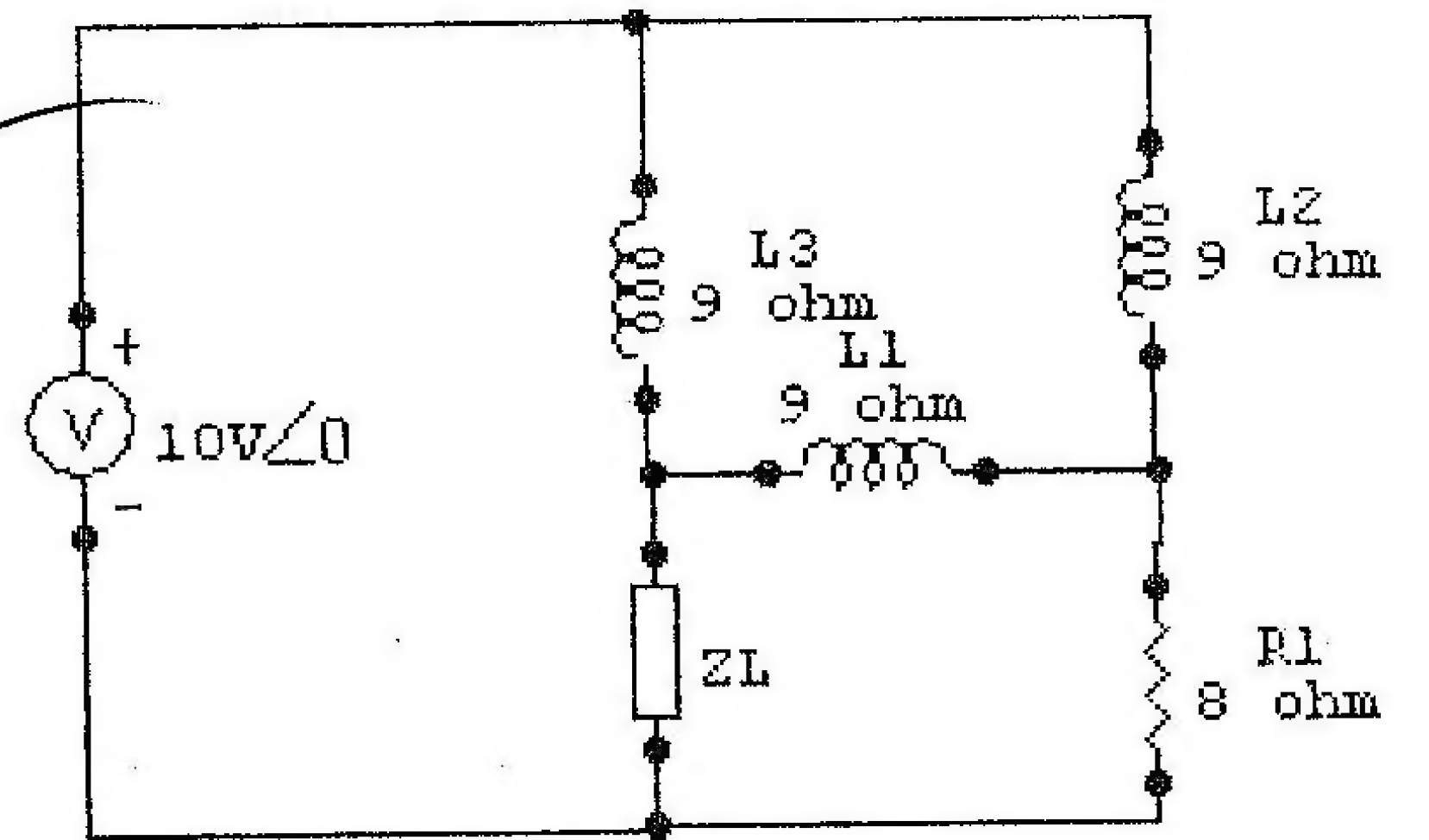
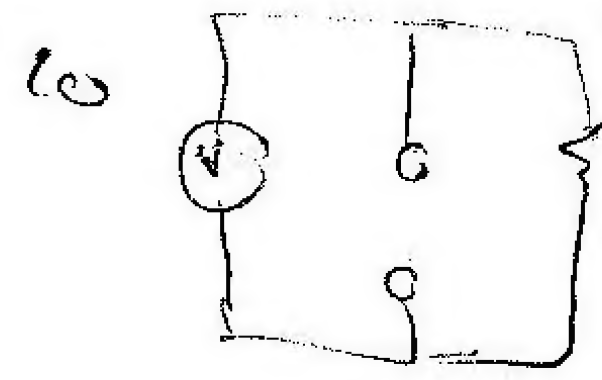
$$\therefore V_c(t) = 25.2 \sqrt{2} \sin(18.2^\circ) = 35.6 \sin 18.2^\circ$$

Q5) Find Z_L for maximum power transfer and find the maximum power

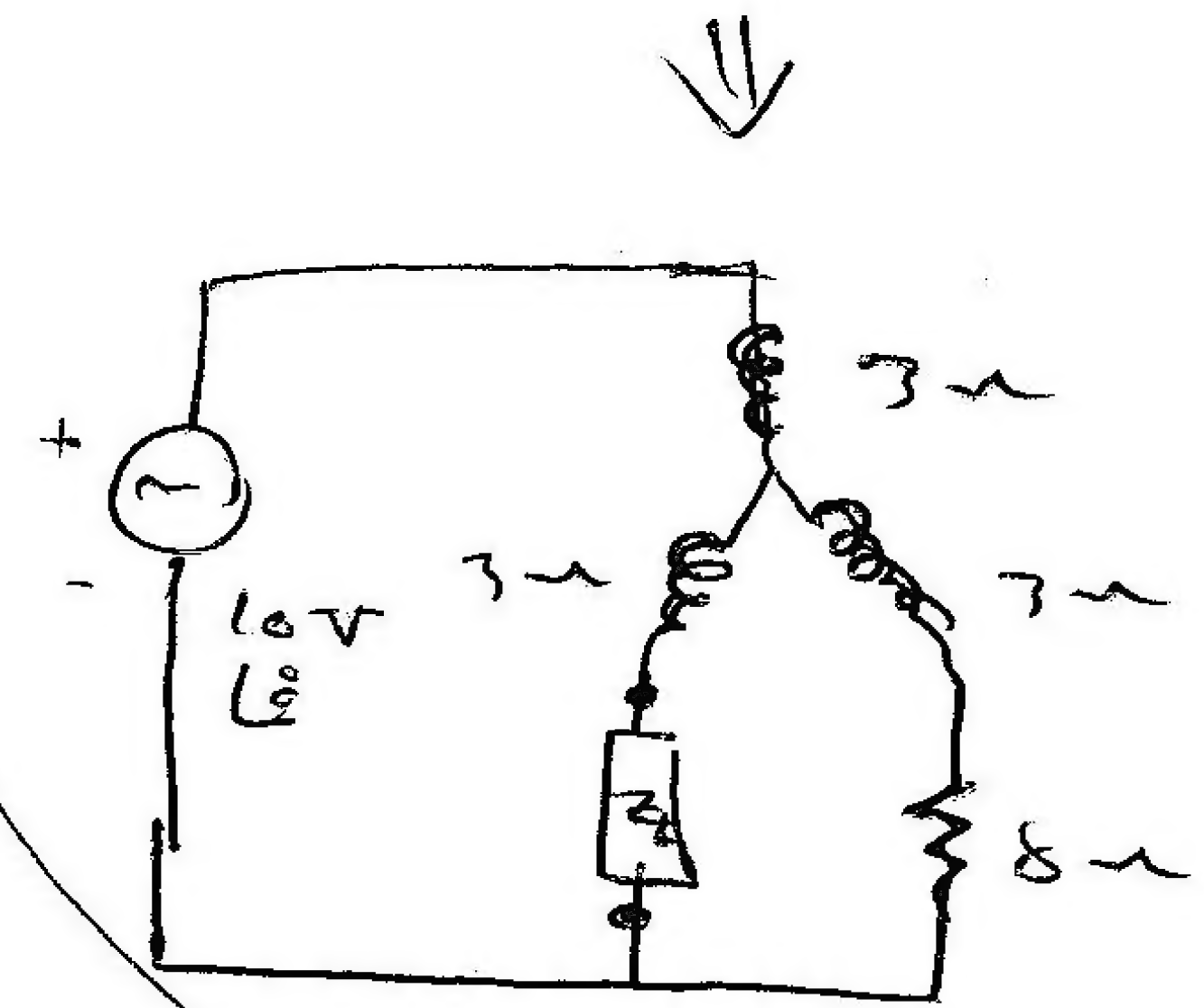
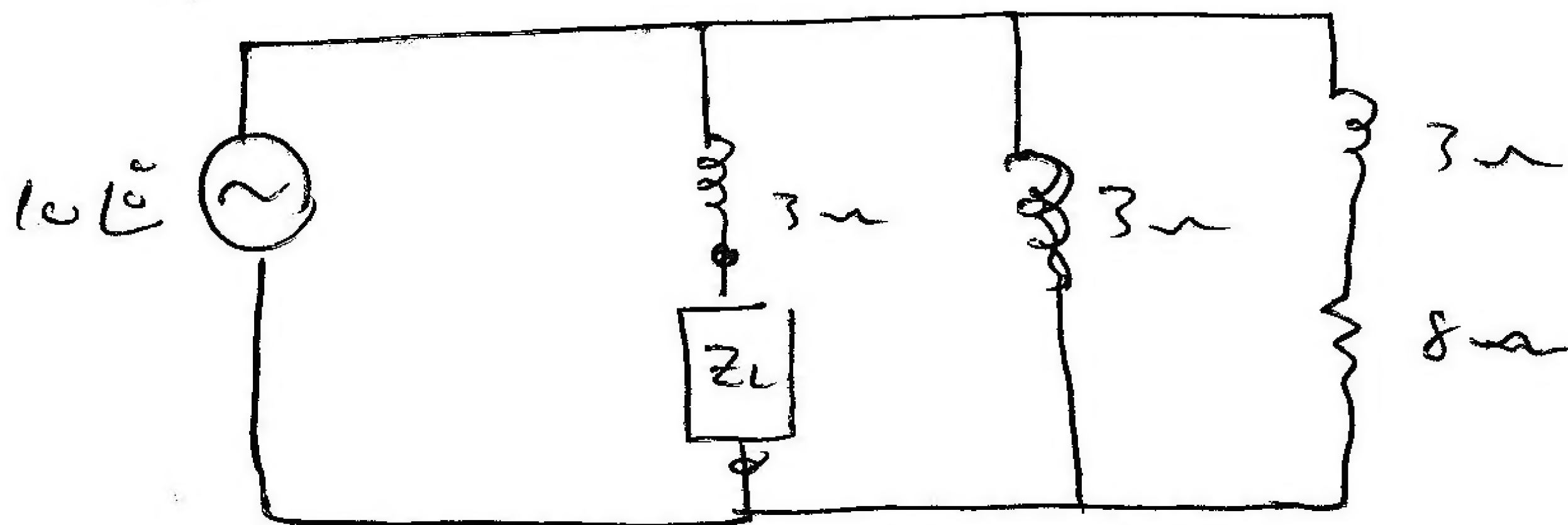
$$Y = \frac{\Delta}{3}$$

(6)

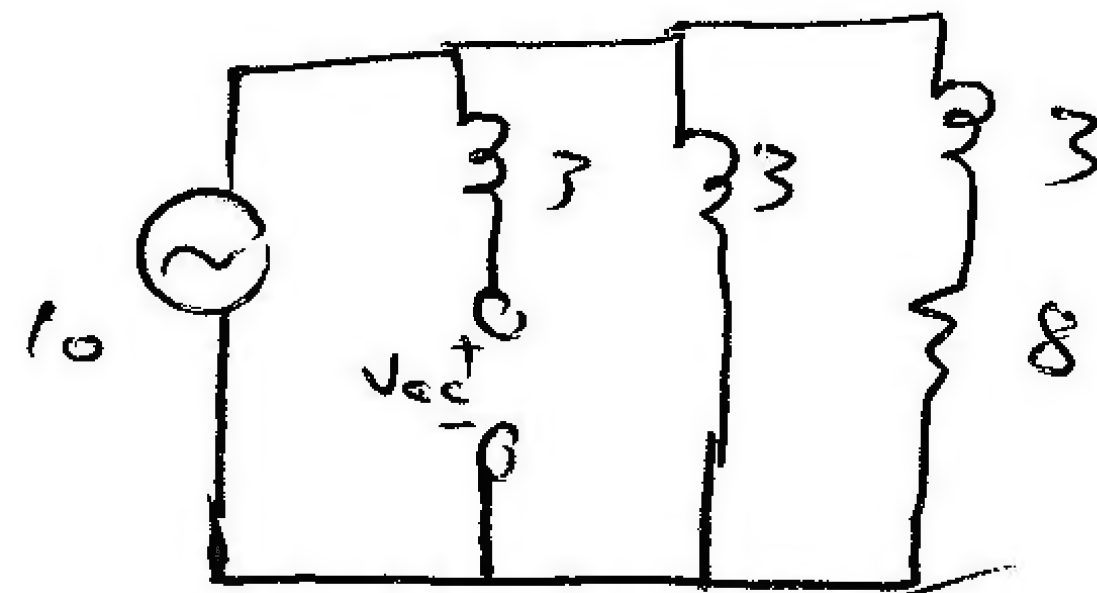
$$Y = \frac{\Delta}{3} = \frac{9}{3} = \boxed{3}$$



\Rightarrow



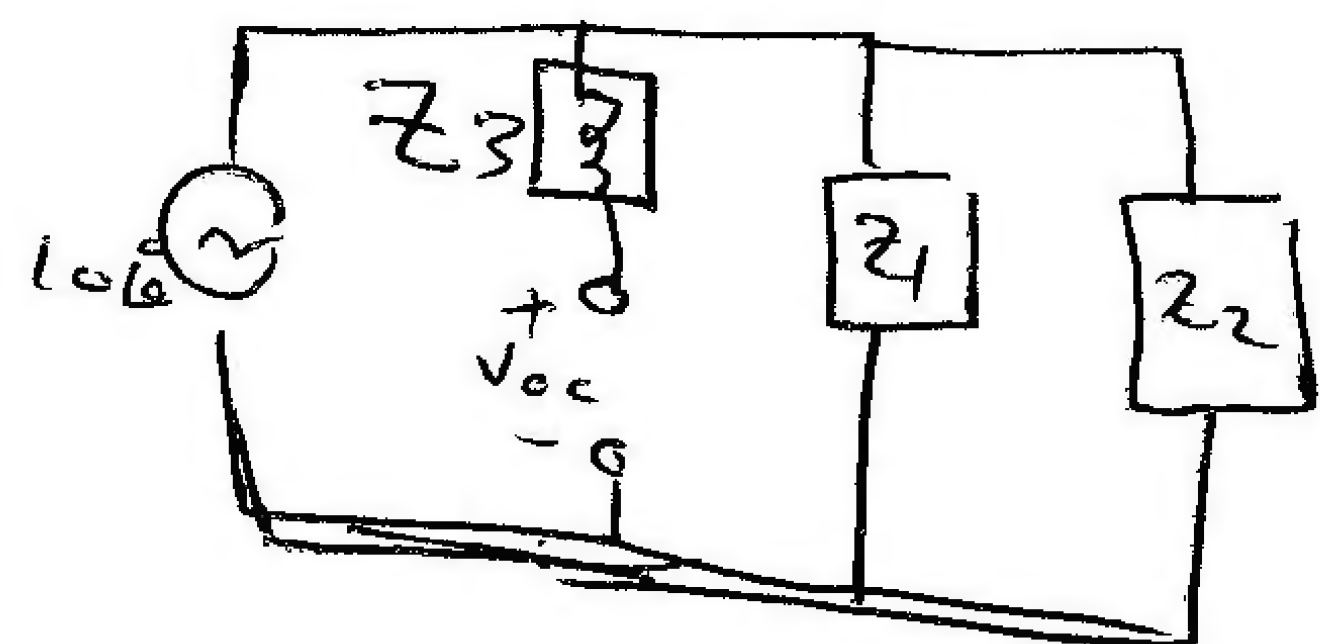
① Find V_{oc}



$$Z_1 = j3 = \boxed{3 \angle 90^\circ}$$

$$Z_2 = 8 + j3 = \boxed{8.5 \angle 20.5^\circ}$$

\Rightarrow



$$\therefore V_{oc} = \boxed{10 \text{ V}} = 10 \angle 0^\circ$$

$$Z_{eq} = \boxed{Z_1 \parallel Z_2 + Z_3}$$

$$Z_1 \parallel Z_2 = \frac{3 \angle 90^\circ \times 8.5 \angle 20.5^\circ}{j3 + 8 + j3} = \frac{25.5 \angle 110.5^\circ}{8 + j6} = \frac{25.5 \angle 110.5^\circ}{10 \angle 36.8^\circ}$$

$$= \boxed{2.55 \angle 73.7^\circ}$$

$$\therefore Z_{eq} = (Z_1 \parallel Z_2) + Z_3 = 2.55 \angle 73.7^\circ + 3 \angle 90^\circ$$

$$= 2.55 (\cos 73.7^\circ + j \sin 73.7^\circ) + j3$$

$$= 2.55 (0.28 + j0.96) + j3$$

$$= 0.714 + j2.448 + j3 = \boxed{0.714 + j5.448}$$

Good Luck
Mahmoud Ahmad

$$\boxed{1} \therefore Z_L \text{ for maximum power transfer} = Z_{eq}^* = \boxed{0.714 - j5.448}$$

2

⇒ Maximum Power transfer

$$= \frac{V_{TH}^2}{4R} = \frac{(10)^2}{4(0.714)} = \frac{100}{2.856}$$

$$\approx 35 \text{ watt}$$